

Frequency Domain Load Calculation for Offshore Wind Turbines (TURBU Offshore)

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ABSTRACT: The design of offshore wind turbines requires to assess a huge amount of different sea-states and wind conditions. Therefore the calculational efficiency of a combined time/frequency domain approach is attractive. This was the reason for the development of the frequency domain tool TURBU Offshore. In addition, such a tool is very feasible for parameter studies; the dynamics of large offshore wind turbines use to be highly sensitive to the natural frequency values. The implementation of TURBU Offshore is based on a modular linear model, with control loops included. The assumptions for structural and aerodynamic modelling are state of the art. A multi-blade transformation for three-bladed rotors eliminates the rotational coupling between the blades and the tower. Two-bladed rotors require a general handling of this coupling, which is however enabled by the model set-up. The program provides the mean loads, power spectra and periodic loads, which are merged to overall load histories for fatigue assessment. Besides, the power spectra make clear the relevance of poorly damped deformation modes. Verification exercises point out that TURBU Offshore works well in stationary conditions and that it predicts damping rates and natural frequencies properly. It is expected that the use of TURBU Offshore in the design of wind turbines will yield substantial cost reduction for the nacelle, rotor and tower. It is recommended to develop guidelines for the complementary use of TURBU Offshore with a time domain tool. The derivation of submodels for control design will be useful, just as the coupling of the linear structural dynamics model to an advanced aerodynamic code.

Keywords: Frequency domain, load calculation, offshore, wind turbines, integrated model.

1. INTRODUCTION

The design of offshore wind turbines significantly differs from that of onshore turbines by the huge amount of different sea-states to which they are subjected. Sea-states are combinations of mean wind speed and direction and significant wave height, period and direction. The large number of potential environmental conditions (sea states) makes it very time consuming to carry out a fatigue analysis with a time domain structural dynamics code like PHATAS [9] in the same straightforward way as for onshore wind turbines. For this reason, a combined time/frequency domain approach for fatigue analysis and sensitivity studies is very desirable: the high calculational speed of the frequency domain tool TURBU Offshore allows for the evaluation of numerous sea-states that are close to a 'centre sea-state', which is assessed with a high-accuracy time domain tool like PHATAS.

Because large offshore wind turbines tend to have low natural frequencies, the combined dynamic excitation by the wind and the waves may have a large impact on the structural integrity of the wind turbine. A frequency domain tool gives a quick insight in (undesired) dynamic interaction and thus allows for efficient parameter study of the choice of natural frequencies and damping.

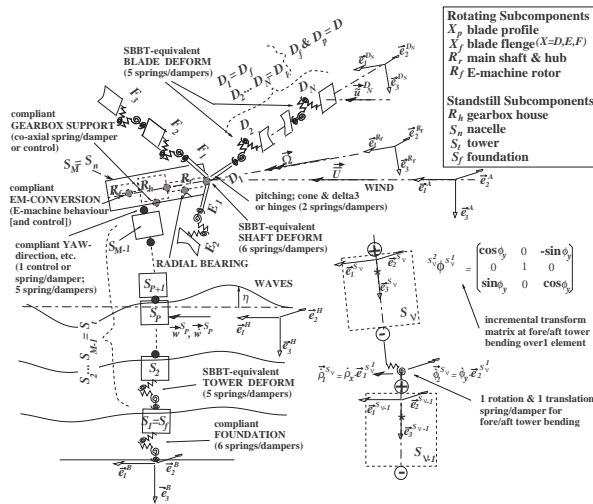
The next sections describe specific topics to be addressed for frequency domain analysis, vizually the derivation of a linear model, the handling of the rotational coupling between the rotor, drive-train and the tower, and the determination of the loads. A summarizing conclusion is subject of the last section.

2. MODEL SET-UP

A linear model of the whole wind turbine is the point of departure for frequency domain analysis. In TURBU Offshore a modularised set-up is obtained by idealising the wind turbine to an assemblage of distinct substructures or *components* – the tower/nacelle, the drive train, and the blades – which in turn consist of one or more discrete, rigid elements (see figure). This modularisation enables to include control loops and to deal with special features like a (free) flapping hinge and dynamic yawing in a well organised and unconstrained way.

Substructures

All flexibility of a substructure is assumed to be concentrated at the attachments between the elements and represented by massless springs and dampers. In general, this approach allows for each element to have up to six degrees of freedom. Usually, the axial deformations are neglected. It may also be useful to assign special generalised degrees of freedom. For example the blade flange element can accommodate flapping and lagging hinges as well as a pitch actuator which is controlled by a linear feedback algorithm. For the same reason, the tower/nacelle substructure can be subdivided into three different sets of elements (*subcomponents*): the foundation, the tower, and the nacelle. The nacelle subcomponent consists of only one element, which can allow for a free or prescribed rotation around the vertical axis thus simulating the behaviour of the yaw control mechanism. In addition it can allow for roll- and tilt-motion oriented angular degrees of



The wind turbine as an assemblage of distinct substructures from elements

freedom as well as the three linear degrees of freedom, which enable to tune the modal frequencies. The bending and distortion of the tower is modelled via a multi-element subcomponent. A one-element foundation subcomponent provides a linear model of the force- and torque-displacement characteristics of soil springs acting perpendicular to the pile and in axial direction. This enables incorporation of the well known tower extension concept for the modelling of a compliant foundation. The drive train substructure consists of three one-element subcomponents: the generator rotor, the gearbox house and the rotor shaft with hub. Via the generator rotor element the control of the generator torque can then be introduced by means of a linear transfer function. The gearbox house can allow for co-axial compliance by means of an angular spring-damper pair or a linearly controlled co-axial support torque. The shaft element is modelled by springs and dampers in the rotor centre. This model allows for bending and distortion and enables to take the effect of the main shaft bearing into account.

The aerodynamic centre of the blade elements is assumed to be located at the quarter chord line. The centre of gravity, the elastic axis and the pitch axis do not necessarily coincide, except at the blade root where the pitch axis point coincides with the elastic axis point. The required cross-sectional data for each element are the mass per unit-length, principal moments of inertia per unit length, bending and shear stiffnesses, the location of the mass centre and elastic axis point, the chord length, the twist angle of the major axes and optionally a flatwise and edgewise pre-bend angle.

The blade *component* is divided into two *subcomponents*, namely the already mentioned blade flange and the blade profile, which includes the blade's structural flexibility. The sequence of the rotations at the blade root side of the blade flange is flapping first, pitching second and lagging last. All blade root rotations, located in the elastic axis point, can have a fixed setting or can be used as degrees of freedom. Fixed settings are defined by the average pitch angle, the cone angle and the δ_3 angle. The 'degree of freedom option' is defined by (i) these fixed settings and on top of that (ii) any linear

transfer function for the pitching torque and constants of the springs and dampers which are placed at the flapping and lagging hinge. The second subcomponent of the blade enables to include structural pitch, blade distortion and flat- and edgewise dynamic deformation. The translation to equivalent spring stiffnesses is done by the program.

Aerodynamic model

To find the equilibrium position of the blades, the aerodynamic forces are computed using Blade Element Momentum (BEM) theory with Prandtl's and Glauert's correction to account for the influence of the wake and corrections to the 2D lift polar to account for 3D-effects. Prandtl's correction is applied twice in order to take into account the absence of a wake in the central part of the rotor. Dynamic effects of the wake, also for yawed conditions, are taken into account by an engineering model as proposed by Schepers and Vermeer [12]. It is also an option to introduce Glauert's correction via an empirical formula for the rotor drag proposed by Wilson [14]81. The inclusion of a dynamic-stall model would require a third subcomponent in the blade model; it will provide an addition to the lift (and drag) coefficient, based on the dynamically processed angle of attack. This feature will be implemented in due course.

The wake model has been implemented rotor-annulus uniform. This means that the aerodynamic forces induced by small oscillations of the blade about the equilibrium position (apparent mass, aerodynamic damping and aerodynamic stiffness) are neglected. A blade-specific wake model would require another subcomponent in the blade model, which has to deal with blade-specific changes in the wake geometry (Prandtl's correction) and the vorticity strength of the wake (the induction factors a and a' and therefore also Glauert's correction).

As the blade-specific changes are neglected, only the transient behaviour of the rotor-annulus uniform induction speeds remains. Such a wake model has been implemented in a separate component, with blade-specific inputs and with equal outputs to the blades.

External forces

The environmental loads to be accounted for include those due to the wind, waves, soil movement, and gravity. Because of the complementary character of a frequency domain tool it was decided to be efficient to take into account only the longitudinal wind speed component. Periodic variations due to wind shear and tower passage are included; the latter via a semi-infinite dipole model. The waves are assumed to have zero-average velocity, i.e. there is no current. Because of the frequency domain approach it is possible to account for the stochastic nature of the wind and the waves.

3. MODELLING PROCEDURE

The (sub)component models of (parts of) the support structure, drive-train and rotor blades are formulated and linked together in such a way that the integrated model is feasible for load calculations. The modelling procedure involves three steps:

1. derivation of linear subcomponent models and linking to component models;

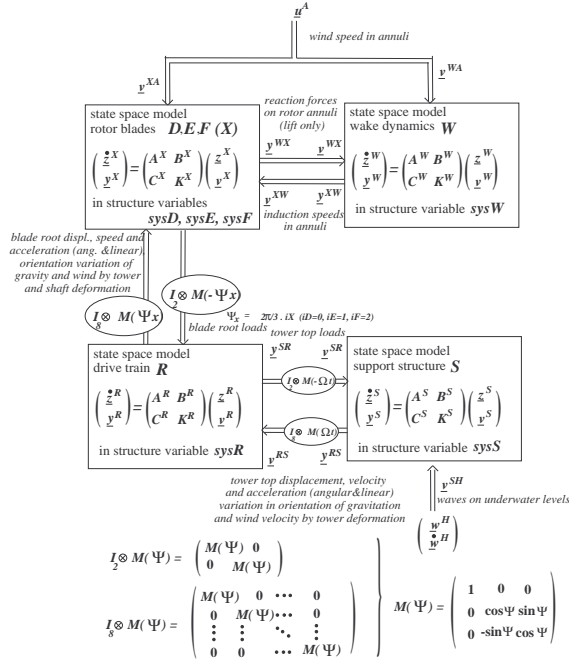


Figure 2: Interdependency of state space models for the distinct substructures of the wind turbine

interface between the drive-train and rotor blades involves the fixed azimuth offset angle of each of the B blades relative to the rotor shaft. The interaction also comprises 'feedforward' of twelve motion and position vectors and 'feedback' of two load vectors.

The interface between each rotor blade and the air tube ('wake') does not include a rotation. The orientation of the axial and tangential reaction forces from a blade on the air tube coincides with the axial and tangential orientation of the air tube in the intersection with that blade. The rotor blades receives the annulus-average axial and tangential induction speed variations.

Handling the rotationally coupled equations of motion

In general, the equations of motion for the integrated model will have coefficients that are periodic in $\bar{\Omega}t$:

$$\begin{aligned}\dot{\underline{z}} &= A(\bar{\Omega}t) \cdot \underline{z} + B(\bar{\Omega}t) \cdot \underline{v} \\ \underline{y} &= C(\bar{\Omega}t) \cdot \underline{z} + K(\bar{\Omega}t) \cdot \underline{v}\end{aligned}$$

This makes them not suitable for well-known solution procedures for systems of ordinary first order differential equations. Because of the polar symmetry of rotors with three or more identical blades a simple transformation – the so-called multi-blade transformation, see Coleman & Feingold [2]– can eliminate the periodic coefficients in the full system equations. The only price to be paid for it consists in modulation of the wind speed variations before they enter the transformed system equations via input vector $\underline{\epsilon}$ and in modulation of the transformed system output variables in $\underline{\eta}$ in order to retransform them along the desired coordinate systems. The following linear time invariant model formulation with modulated input preprocessing and output

postprocessing then applies:

$$\begin{aligned}\underline{\epsilon} &= T_{v_{cm}}^{-1}(\bar{\Omega}t) \cdot \underline{v} \\ \dot{\underline{q}} &= A_{cm} \cdot \underline{q} + B_{cm} \cdot \underline{\epsilon} \\ \underline{\eta} &= C_{cm} \cdot \underline{q} + K_{cm} \cdot \underline{\epsilon} \\ \underline{y} &= T_{y_{cm}}(\bar{\Omega}t) \cdot \underline{\eta}\end{aligned}$$

The invariant input, state transition, output and feedthrough matrix in this model is obtained from the corresponding periodic matrices in the earlier mentioned model via the Coleman transformation matrix $T_{z_{cm}}(\bar{\Omega}t)$ on the state vector, by performing the transformation for an arbitrary fixed value of the rotor azimuth angle $\bar{\Omega}t$. The Coleman transformation involves the mapping of any corresponding quantities on the rotor blades D , E and F to multi-blade coordinates via the 3×3 matrix kernel

$$\begin{pmatrix} \frac{1}{3} & & \\ \frac{2}{3} \sin(\psi_r) & \frac{1}{3} & \\ \frac{2}{3} \cos(\bar{\Omega}t) & \frac{2}{3} \sin(\bar{\Omega}t + \frac{2}{3}\pi) & \frac{1}{3} \\ \frac{2}{3} \cos(\bar{\Omega}t) & \frac{2}{3} \cos(\bar{\Omega}t + \frac{2}{3}\pi) & \frac{2}{3} \cos(\bar{\Omega}t + \frac{4}{3}\pi) \end{pmatrix}.$$

It also involves the mapping of x , y and z -coordinates of any vector on the rotor shaft to multi-blade coordinates via the 3×3 matrix kernel

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\bar{\Omega}t) & -\sin(\bar{\Omega}t) \\ 0 & \sin(\bar{\Omega}t) & \cos(\bar{\Omega}t) \end{pmatrix}.$$

All input and output variables as well as all state variables are accompanied by unique signal names. Each signal name involves the identifier for the destination, source or possessing component for an output, input or state variable respectively. This enables to create in an automated way the appropriate matrix kernels on the right locations in the Coleman transformation matrices for each set of 3 blade variables, each rotor shaft vector variable and each support structure variable.

It should be noted that periodic coefficients due to external forces (gravity and uniform wind loading) are not eliminated by the Coleman transformation. Also the periodic coefficients in the equations of motion of single- and two-bladed rotors cannot be eliminated by this transformation because the inertia of the rotor disc changes with the angular position, i.e. the rotor does not possess polar symmetry. Therefore a more general transformation method like the harmonic balance method or the Floquet Transition Matrix method has to be used, see Dugundji & Hultgren [3], Miller *et al* [10], Wendell [13], Peters and Hohenemser [11], Friedmann [4].

The parametric excitations terms due to gravity and wind loads can generally be ignored for all but extremely flexible blades.

Acceleration of the dynamic analysis.

An approach to make the dynamic analysis of the complete structure more numerically efficient is a method proposed by Graig and Bampton [6]. The method is closely related to a method proposed by Hurty [7], [8], known as *component-mode synthesis*, and a similar method developed by Gladwell [5], generally referred to as *branch-mode analysis*. The

method allows to significantly reduce the degrees of freedom in the blade and tower model without loss of accuracy in the dynamic behaviour of the lower bending modes; the number of modes to be included can be specified.

4. CALCULATION OF LOADS

The calculation of loads in the frequency domain involves the transformation of the power spectrum and coherence function of the wind turbulence and waves to the power spectra of forces and torques. In addition the (almost) periodic loads are to be determined from the azimuth dependent gravity loading and wind speed variations caused by wind shear, oblique inflow and tower shadow. These loads, together with the mean loads, are linked together in order to assess the fatigue loading, usually via rainflow counting.

Stochastic loads from turbulence and waves

The required input spectrum matrix S pertains to the wind turbulence \tilde{u} as experienced on the rotating blades and the wave speed w and acceleration \dot{w} . As a logical consequence of the Blade Element Momentum theory, the turbulence is considered in the intersections of the blades with a discrete number of rotor annuli (P). The hydrodynamic conversion is also discretised; each of the N_{uw} underwater elements of the support structure is linked to a water segment in which identical wave behaviour is assumed.

The stochastic consideration of the longitudinal turbulence for the average rotor speed $\bar{\Omega}$ yields a $B \cdot P \times B \cdot P$ spectrum matrix that accounts for the rotational sampling effect. For the waves a $2 \cdot N_{uw} \times 2 \cdot N_{uw}$ spectrum matrix results, which includes the correlation between the horizontal wave speed and acceleration. The overall input spectrum matrix looks like:

$$S_{\underline{v}\underline{v}}(\omega) = \begin{pmatrix} S_{\tilde{u}\tilde{u}}(\omega) & 0 \\ 0 & S_{\tilde{w}\tilde{w}}(\omega) \end{pmatrix}$$

The modulation of inputs and outputs that is involved with the Coleman transformed model representation makes the output vector a so called cyclo stationary process. This implies that its covariance function matrix is periodic in the rotational frequency ($T = 2\pi/\bar{\Omega}$). In that case, the spectrum searched for is the Fourier transform of the *period-average* covariance function \tilde{C} (superscript \dagger stands for complex conjugate transpose):

$$S_{\underline{y}\underline{y}}(\omega) = \int_{-\infty}^{\infty} e^{-j\omega\tau} \cdot \tilde{C}_{\underline{y}\underline{y}}(\tau) d\tau$$

with

$$\tilde{C}_{\underline{y}\underline{y}}(\tau) = \frac{1}{T} \int_0^T \mathbb{E} [\underline{y}(t + \tau) \cdot \underline{y}^\dagger(t)] dt$$

The transfer function matrix H in the frequency domain representation of the Coleman transformed model

$$\underline{\eta}(\omega) = H(\omega) \cdot \underline{\epsilon}(\omega)$$

with:

$$H(\omega) = C_{cm} (j\omega \cdot I - A_{cm})^{-1} \cdot K_{cm},$$

together with the coefficient matrices \hat{U}_m and $\hat{\Gamma}_n$ in the Fourier series formulation of the input and output modulation

$$\underline{\epsilon}(t) = \sum_{m=-1}^1 \hat{U}_m \cdot e^{j \cdot m \bar{\Omega} t} \cdot \underline{v}(t)$$

$$\underline{y}(t) = \sum_{n=-1}^1 \hat{\Gamma}_n \cdot e^{j \cdot n \bar{\Omega} t} \cdot \underline{\eta}(t),$$

are the building blocks for mapping the input spectrum to the output spectrum:

$$S_{\underline{y}\underline{y}}(\omega) = \sum_{n=-1}^1 \sum_{m=-1}^1 \sum_{\nu=-1}^1 \sum_{\mu=-1}^1 \hat{\Gamma}_n \cdot H(\omega - n\bar{\Omega}) \cdot \hat{U}_m \cdot S_{\underline{v}\underline{v}}(\omega - (n+m)\bar{\Omega}) \cdot \hat{U}_\mu^\dagger \cdot H^\dagger(\omega - \nu\bar{\Omega}) \cdot \hat{\Gamma}_\nu^\dagger$$

with all combinations of n, m, ν and μ excluded for which $n + m - \nu - \mu \neq 0$.

Periodic loads from gravitation, wind shear, tower shadow and oblique inflow

The periodic gravitation influence on the rotor blades are modelled via a 3-coordinate vector per blade with non-zero periodic coordinate values in the span- and leadwise direction of the blade root ($g_{\text{per}}^{D_0}$ for blade D). The periodic wind speed variations will differ over the blade elements and thus require a specific vector per blade element. Thus the periodic wind speed variations are modelled in a large vector per blade with $3 \cdot N$ coordinates ($\underline{u}_{\text{per}}^D$ for blade D). The overall periodic input vector is the 'stack' of the blade individual periodic gravitation and wind speed vectors and can be written as Fourier series with coefficient vectors $\hat{\underline{v}}_\ell$ of dimension $B \cdot 3 \cdot (N + 1)$. The periodic input vector looks like:

$$\underline{v}_{\text{per}}(t) = \begin{bmatrix} g_{\text{per}}^{D_0}(t) \\ \underline{u}_{\text{per}}^D(t) \\ g_{\text{per}}^{E_0}(t) \\ \underline{u}_{\text{per}}^E(t) \\ g_{\text{per}}^{F_0}(t) \\ \underline{u}_{\text{per}}^F(t) \end{bmatrix} = \sum_{\ell=-L}^L \hat{\underline{v}}_\ell \cdot e^{j \cdot \ell \bar{\Omega} t}$$

The required modulation for the input vector of the Coleman transformed model yields a Fourier series with coefficients $\hat{\underline{\epsilon}}_k$ ranging from $-L - 1$ to $L + 1$:

$$\underline{\epsilon}_{\text{per}}(t) = \sum_{k=-L-1}^{L+1} \hat{\underline{\epsilon}}_k \cdot e^{j \cdot k \bar{\Omega} t}$$

with

$$\hat{\underline{\epsilon}}_k = \sum_{m=\max(-1, k-L)}^{\min(1, k-L)} \hat{U}_m \cdot \hat{\underline{v}}_{k-m}$$

The desired periodic output variables are obtained as a Fourier series with coefficient vectors $\hat{\underline{y}}_p$ that range from $-L - 2$ to $L + 2$. The determination of $\hat{\underline{y}}_p$ requires the availability of the Fourier coefficient vectors $\hat{\underline{\eta}}_k$ of the Coleman model output variables. The latter follow straightforward from the

input Fourier coefficients $\hat{\epsilon}_k$ via the transfer function matrix H in the very frequency $k\Omega$ for $k = p - n$:

$$\underline{y}_{\text{per}}(t) = \sum_{p=-L-2}^{L+2} \hat{y}_p \cdot e^{j \cdot p \Omega t}$$

with

$$\hat{y}_p = \sum_{n=\max(-1, p-L-1)}^{\min(1, p-L-1)} \hat{\Gamma}_n \cdot H((p-n)\Omega) \cdot \hat{\epsilon}_{p-n}$$

Assessment of the fatigue loading

The usual way to assess the fatigue loading occurs via rain-flow counting on load histories, which results in load histograms. The load histories are obtained from the mean values, power spectra and Fourier series for the periodic loads. Stochastic time series are generated from the power spectra via a so called realisation algorithm.

The generation of contemporary load histories on different spots or in different directions in a cross section requires a realisation algorithm that also accounts for the cross power spectra between the loads. These cross power spectra are available via the off-diagonal elements of the output spectrum matrix. Such a multivariable realisation algorithm is also used for the generation of stochastic wind fields, e.g. as implemented in the computer program SWIFT [15].

Next to this approach, direct translation of power spectra into load histograms is possible. This is common practice in conventional offshore engineering, in which the fatigue governing loads are purely stochastic. Algorithms are e.g. provided by Dirlik and examined for application on wind turbines [1]. If however the mean and periodic loads have considerable influence on the fatigue damage, it is recommended to apply this approach carefully.

5. RESULTS

Initial load calculations have been performed on the so called OptiOwecs Windturbine, a 3MW offshore wind turbine with 3 blades, variable speed and pitch control. The hubheight amounts to 60 m above sea level; the rotor diameter amounts to 80 m. A PhD study on design methods for offshore wind turbines used the OptiOwecs wind turbine as a typical offshore wind turbine [16]. Figure 3 shows the power spectra of stochastic components of the blade root flap and lead moment and of the tower bottom fore-aft and sideward moment. The waves are in fore-aft direction and are derived from the Pierson Moskowitz spectrum. The wind speed variations obey the Kaimal spectrum and coherence function. Detailed comparisons with the time-domain data obtained with the DUWECS program [17] will be reported in a subsequent paper on frequency domain load calculation with TURBU Offshore.

6. CONCLUSION

TURBU Offshore is a computer program for frequency domain analysis of wind turbines with three blades and is based on a first order modularised state space model of the linearised system dynamics. The model is derived in a straightforward way from aerodynamical and mechanical laws for conservation of impulse. The model set-up enables

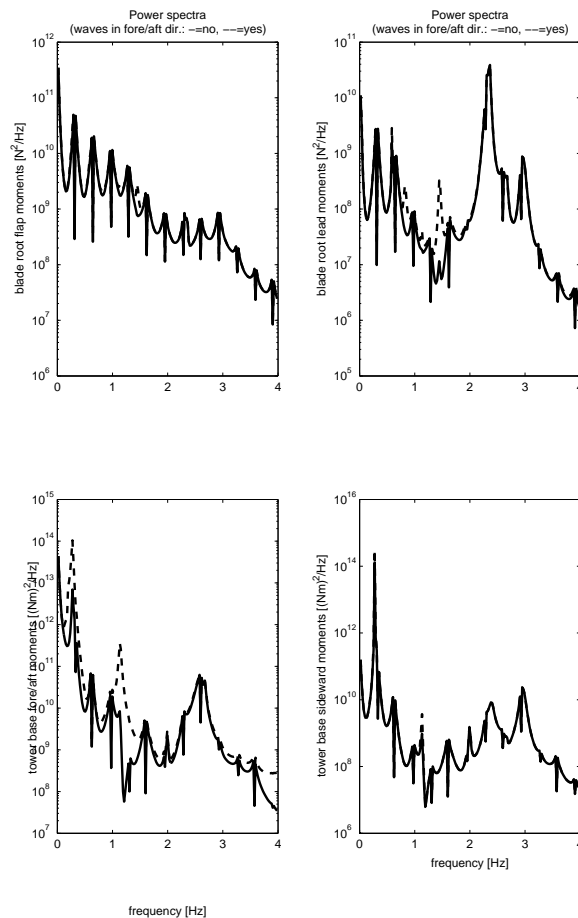


Figure 3: Power spectra from TURBU Offshore; with and without hydrodynamic loading

to include control algorithms and generalised degrees of freedom in a convenient way. The rotational coupling between the rotating blades and the tower is eliminated via a multi-blade coordinate transform. This ‘modulated coupling’ is ‘moved to the edges’ of the model: the model inputs are to be multiplied with harmonic functions before they enter the transformed model; the model outputs have to be multiplied with harmonic functions in order to obtain the real-life load and motion variables.

TURBU Offshore is also feasible for aero-elastic stability analysis and control design. The incorporation of control loops in the program modules of the rotor blades and drivetrain makes it very valuable for realistic predictions on the stability. Since the size of the wind turbine is still increasing, the natural frequencies fall down and thus interfere more and more with the bandwidth of control loops.

The program output for the stochastic behaviour consists of the power spectra of the loads on the blades and support structure. These are transformed into time series for the stochastic loads with a realisation algorithm. In addition, the mean values of the loads and the Fourier series for the periodic loading are computed. The overall load histories are

assembled and subject to rainflow counting. The resulting load histograms are used for fatigue assessment in load cases.

Especially the power spectra provide a clear view on the relevance of poorly damped deformation modes of the blades, support structure or whole system on the fatigue loading, since this heavily depends on the rate of excitation of such modes. This very excitation, which is sized by the power spectrum of wind and waves, the coherence of turbulence in the rotor plane and the rotational speed, is catered for by the power spectra. Conventional aero-elastic stability analysis does not take into account the excitation of the deformation modes.

The integrated linear model also is the base for advanced control design at ECN; see also [18].

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